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## СИСТЕМНЫЙ АНАЛИЗ, МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ

### METHODS OF WEIGHTS DEFINITION IN MULTICRITERIA ANALYSIS

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Main article purpose – the analysis of the advantages and shortcomings for various methods of weight coefficients (WC) determination for factors sets, considered with multicriteria decision making. Five methods are compared: method of cards; AHP (Analytical Hierarchy Process); Direct weighting; Fixed point scoring; MACBETH (Measuring Attractiveness by Categorial Based Evaluation Technique). The last method allows to combine two types of information: about preferences of the persons, which making decisions; about importance of criteria and their interactions, limited in couples of criteria. Authors emphasize, that expedient for application of the scheme of WC can depend of the chosen method. For methods of cards; AHP, MACBETH in article are given the concrete examples of their application, which illustrate technique of weight coefficients calculation.

**Keywords:** multi-criteria analysis, weight schemes, aggregation; hierarchisation, method of cards, method AHP, method MACBETH

### МЕТОДЫ ОПРЕДЕЛЕНИЯ ВЕСОВ В МНОГОКРИТЕРИАЛЬНОМ АНАЛИЗЕ

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Основная цель статьи – анализ преимуществ и недостатков различных методов определения весовых коэффициентов (ВК) для совокупностей факторов, учитываемых при принятии многокритериальных решений. Сравнены пять методов: метод карт (Method of the cards); АНР (Analytical Hierarchy Process); прямого взвешивания (Direct weighting); оценки неподвижной точки (Fixed point scoring); МАСВЕТН (Measuring Attractiveness by Categorical Based Evaluation Technique). Последний метод позволяет комбинировать два типа информации: о предпочтениях лица, принимающего решения; о важности критериев и их взаимодействиях, ограниченной парами критериев. Авторы подчеркивают, что целесообразные для применения схемы ВК могут зависеть от выбранного метода. Для методов «карт»; АНР, МАСВЕТН в статье приведены конкретные примеры их применения, иллюстрирующие технику вычисления весовых коэффициентов.

**Ключевые слова:** многокритериальный анализ, весовые схемы, агрегация; иерархизация, метод карт, метод АНР, метод МАСВЕТН

**Introduction.** In project management the consequences, associated with procedures weights, are significant - because these occur for the most parts in the decision making process of companies. The calculated weights often used to size the project to focus efforts and investments towards the most important objectives. In multicriteria analysis, the knowledge of the weights is essential. Determining the importance of a criterion over another is a major issue for both scientific and political. This is what then justifies the interest of a research on the methods of weighting. Therefore in the literature exists a significant variety of weighting methods. These methods have been the subject of several studies, especially in the field of project management, where will, for example, choose the best contractors, which can run the project [1, 9, 13], in the field of environmental impact assessment, social and territorial [16]. These methods have also proved their worth in the design and optimization of energy systems in the assessment of the environmental dimension of land and also in the calculation of nitrogen emissions on a group of farms [17, 22]. Other work on the assessment of appropriate measures against flooding, the rural water supply and sanitation in developing countries have relied on these methods [3]. In it a literature review have been conducted to identify methods for determining the weights, that would be likely to respond to different research objectives. The purpose of this article was a literature review of methods for determining the weight schemes in multicriteria analysis.

**1. Method of the cards.** This method was proposed by [23] to weigh multiple criteria in an environmental context. The main idea of the method of the cards (Simos, [8]), is to order the criteria and specify the «gap» of importance between two criteria. It provides the expert two types of cards: those representing criteria (one card per test), and white cards [12]. As a first step, the expert must order the «criteria» cards in order of importance (ex aequo authorized). Then he can put white card between cards criteria, with significance as the number of white cards between two criteria - the greater the difference in importance between these two criteria, is large. We propose to present a structured way of this method: first highlight the difference between ordinal and cardinal rankings; then we outline the principles for the establishment of a cardinal ranking of an ordered set by this method and those of weights determination from this cardinal ranking.

*1.1. Ordinal classification, cardinal classification.*

The two terms «ordinal» and «cardinal» correspond to two possible definitions of natural numbers.

✓Ordinal integer denotes the position of a term in an ordered series of elements of a set on. In the sequence (A, B, C, D), the position C is designated by the integer 3.

✓The cardinality of a set is the integer elements of this set. This term quantitative consonance. The cardinality of the set {a, b, c} is 3.

*Example of classification by the method of the cards*

Suppose we want to classify 8 products, obtained in a series production. We submit the eight products to a test consisting of 20 consecutive tests of equal importance for the appreciation of product quality. For each of these tests, it gives the result: yes (success) or not (failure). Let us study the ordinals and the cardinals of the undergone tests: this is called the note,  $n(\mathbf{P})$  taking values in  $\{0, \dots, 20\}$ . We classify the products in the range - for example in descending order of notes, and is thus obtained the classification  $c(\mathbf{P}_i)$ , giving the product an ordinal value in the set  $\{1, \dots, 8\}$ . Thus we have defined a cardinal ranking, that of the notes and an ordinal ranking of the rank. Notes products and their classification are for example:

Product  $\mathbf{P}_1 \mathbf{P}_2 \mathbf{P}_3 \mathbf{P}_4 \mathbf{P}_5 \mathbf{P}_6 \mathbf{P}_7 \mathbf{P}_8$   
 Note on 20 12 11 04 19 09 13 07 05  
 Ranking 3 4 8 1 5 2 6 7

We can generalize this example, considering a product  $\mathbf{P}$  extract a set, that contains  $\mathbf{n}$ . Were measured on each product a given physical quantity will result. Choosing a multiple or submultiple of the unit, it is always possible to associate a measure of this quantity  $\mathbf{S}$  as an integer - for example the length of a car can be considered a whole number of millimeters, the temperature of a medium - an integer of kelvins. The measure  $\mathbf{S}$  forms a cardinal product ranking: Cardinal ranking of  $\mathbf{P}$  is the integer value  $\mathbf{s}$ . It takes its values in  $\mathbf{N}$  (natural integers). If one classifies the products according to the value of their exit, the product  $\mathbf{P}$  will have a rank  $c(\mathbf{P})$  in the family: thus form an ordinal ranking that takes values in  $\{1, \dots, \mathbf{n}\}$ . In case of ex æquo, given the two elements, the same rank and the following unit is shifted. We can deduce an ordinal ranking of a cardinal ranking: knowing the numerical values of the cardinal size for each element of the set, you can sort them by ascending or descending order of the value and derive an ordinal ranking of the elements. The reciprocal is obviously false - you can not deduct notes products on their classification. In particular, by saying that the  $\mathbf{P}_4$  product is first,  $\mathbf{P}_6$  second and  $\mathbf{P}_7$  sixth, that does not mean that  $\mathbf{P}_6$  is twice less good than  $\mathbf{P}_4$ , than  $\mathbf{P}_7$  is six times less good than  $\mathbf{P}_4$  or that  $\mathbf{P}_7$  is three times less good than  $\mathbf{P}_6$ .

*1.2. Description of the method of the cards on the example.*

When passing notes in the standings (see Fig. 1), there is obviously a loss of information. However, an agent, which knows well the eight products of the previous example, is able at the time when it defines the twenty tests, to produce an estimated ranking and to even consider the notes, probable that the products will obtain. For this, the mental process is similar to the positioning of products on a graduated scale, the distances separating the products being the image from the foreseeable variation of the notes. This remark is illustrated in Figure 2.

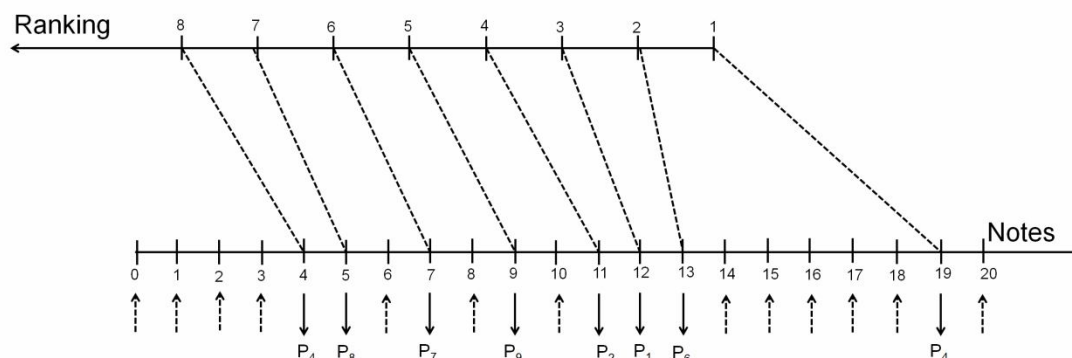


Fig. 1 . Diagram classification of products by the method cards

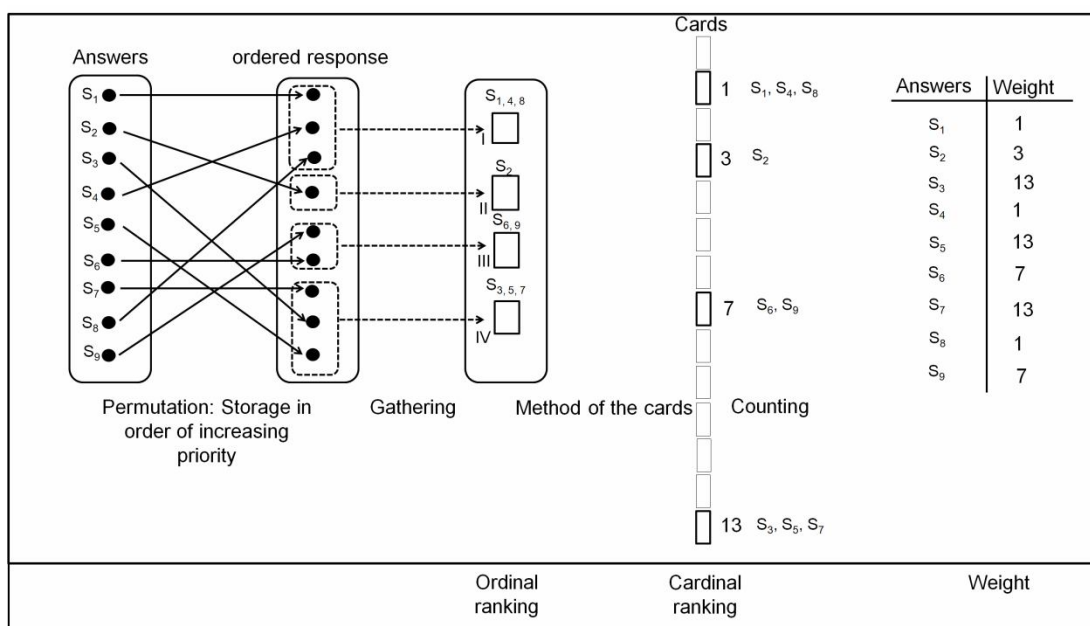


Fig. 2 . Diagram illustration of the determination of the weight of nine objectives by the method cards

In particular, the agent distinguishes well excel it then produces  $P_4$ , which precedes the other products clearly, comes a group from products around or slightly to the top of the average (in the order  $P_6$ ,  $P_1$ ,  $P_2$  and  $P_5$ ) and finally three products pretty significantly below ( $P_7$ ,  $P_8$  and  $P_3$ ). Figure 1 shows the ranking of the eight products on the top axis and notes on the lower axis. The significance of the arrows will be given later. Let us imagine that the agent has in hand eight index cards, each relating to a product with a manufacturing code. To pass from the ranking, estimated with the probable notes, it can use a stock of blank index cards, and insert as he wants, between or after those products, with the only constraint to have 21 records on hand at the end. We call card, a printed or virgin sheet. Is thus obtained the stack of 21 cards, as shown in Figure 1, the product sheets are represented by dashed arrows upward. Let us give some objective arguments that the agent can use:

- ✓  $P_8$  and  $P_3$  products are bad, but not zero: place four cards for example before them.
- ✓ The product  $P_4$  certainly good, but not perfect, do not pass, i.e. can not be successful in all tests: therefore placed a card behind him.
- ✓ Products  $P_6$ ,  $P_1$  and  $P_2$  are of good quality, but are several lengths of  $P_4$ : we place e.g. five cards between  $P_4$  and group.
- ✓ We pass from product  $P_2$  to  $P_5$ ,  $P_5$  to  $P_7$  and  $P_7$  to  $P_8$  by small successive jumps substantially equal: placing a card every time between them.

We have described on this example, the method of the cards. It makes it possible to define a cardinal ranking of  $N$  objects from only ordinal ranking.

Let us now give a synthesis of these principles. Figure 2 gives a complete illustration of different steps of the process for determining the weights of  $N = 9$  objectives associated with the outputs  $S_1, \dots, S_9$ . The  $N$  objects are registered on cards. Those are classified in the order, defined by the ordinal ranking. It should well be understood, that the cards are only one mediator of the human intuition. The procedure consists of set usage of blank cards and:

- ✓ to place before the first card a number of blank cards, representative of the difference between this object and the imaginable bad object;

✓to place after last the card-object a number of blank cards, representative of the difference between this object and the best imaginable object;

✓to intercalate between the card-objects a representative number of cards, separating the two consecutive objects.

One thus obtains a cardinal ranking on a scale going of zero until the full number of cards least one unit (20 in the example).

In summary, the method of the cards is simple and easy to use. But the obtained weighting will not reflect systematic differences in importance between criteria. Moreover, it appears too intuitive to us [9,7].

**2. Determination of weights in the AHP method.** The pairwise comparison method, called AHP (Analytical Hierarchy Process), was developed by [19] and widely used since. This method breaks up into four steps: hierarchisation of the criteria by importance - of most important at least important; construction of matrix, starting from the comparison two to two of the criteria; determination of the weights, associated with each criterion thanks to the approximate method of calculating of the eigenvectors; finally - checking of consistency of the result.

*2.1. Hierarchisation of the criteria by importance.*

Let  $C_1 \dots C_i \dots C_n$  - the set of criteria for which the weights are desired. The prioritization must lead to a ranking in which  $C_1$  is greater than  $C_{i-1}$  is greater than  $C_i$  and so on until  $C_n$  be the criterion of less importance. The relation of importance, defined here, is not strict. That means that  $C_{i-1}$  is less important or more important than  $C_i$ .

*2.2. Comparison in pairs of criteria.*

Let  $W_i$  the weight of criterion  $C_i$ . The pairwise comparison of the criteria leads to define the degree importance of a criterion relative to the other – in accordance with Table 1. A scale of values from 1 to 9 is adopted and used to introduce the judgments of the decision maker closer to reality [10].

Table 1

**Judgment scale of relative importance for pairwise comparison[18]**

Intensity of Importance	Definition	Explanation
1	Equal importance	Two activities contribute equally to the objective
3	Moderate importance	Experience and judgment slightly favour one activity over another
5	Strong importance	Experience and judgment strongly favour one activity over another
7	Very strong or demonstrated importance	An activity is favoured very strongly over another, its dominance demonstrated in practice
9	Extreme importance	The evidence favouring one activity over another is of the highest possible order of affirmation
2, 4, 6, 8	For interpolation between the above values	Sometimes one needs to interpolate a compromise judgment numerically because there is no good word to describe it

For example, if the criterion  $C_i$  – essential importance relative to the criterion  $C_j$ , then the ratio  $w_i/w_j$  will be equal to 5. Comparing between them each criterion, the following matrix is obtained:

$$A = \begin{bmatrix} a_{11} & \dots & a_{1i} & a_{1j} & \dots & a_{1n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{i1} & \dots & a_{ii} & a_{ij} & \dots & a_{in} \\ a_{j1} & \dots & a_{ji} & a_{jj} & \dots & a_{jn} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & \dots & a_{ni} & a_{nj} & \dots & a_{nn} \end{bmatrix} \quad \text{with } a_{ij} = \frac{w_i}{w_j} \text{ and } a_{ii} = 1 \quad (1)$$

where  $a_{ij}$  is the intensity of the importance of  $C_i$  to  $C_j$ ;  $w_i$  – the weight, associated with  $C_i$ ;  $A$  – the matrix of judgments.

This square matrix is reciprocal because  $a_{ij} = \frac{w_i}{w_j} = \frac{1}{a_{ji}}$ . Thus, the use of reciprocal

values for the positions of transition, reduces the number of comparisons to  $n(n-1)/2$ .

### 2.3. Determination of weights, associated with each criterion.

We propose two approaches for determining weights: approach, based on the normalization of the matrix; the approach, based on eigenvalue analysis.

#### Normalization method

After the construction of matrix  $A$ , one searches the vector of the weighting coefficients. One divides each  $a_{ij}$  by the sum of values of the corresponding column. It is said that normalizes the matrix and normalization then allows meaningful comparisons between elements. Then an average is performed by lines: all elements of a row of the normalized matrix are added together and then divided by the number of entries it contains. This mathematical operation is governed by the equation (1):

$$w = \begin{bmatrix} \frac{\frac{a_{11}}{\sum_{k=1}^n a_{k1}} + \dots + \frac{a_{1i}}{\sum_{k=1}^n a_{ki}} + \dots + \frac{a_{1n}}{\sum_{k=1}^n a_{kn}}}{n} \\ \dots \\ \frac{\frac{a_{i1}}{\sum_{k=1}^n a_{k1}} + \dots + \frac{a_{ii}}{\sum_{k=1}^n a_{ki}} + \dots + \frac{a_{in}}{\sum_{k=1}^n a_{kn}}}{n} \\ \dots \\ \frac{\frac{a_{n1}}{\sum_{k=1}^n a_{k1}} + \dots + \frac{a_{ni}}{\sum_{k=1}^n a_{ki}} + \dots + \frac{a_{nn}}{\sum_{k=1}^n a_{kn}}}{n} \end{bmatrix} = \begin{bmatrix} \frac{\sum_{l=1}^n \left[ \frac{a_{1l}}{\sum_{k=1}^n a_{kl}} \right]}{n} \\ \dots \\ \frac{\sum_{l=1}^n \left[ \frac{a_{il}}{\sum_{k=1}^n a_{kl}} \right]}{n} \\ \dots \\ \frac{\sum_{l=1}^n \left[ \frac{a_{nl}}{\sum_{k=1}^n a_{kl}} \right]}{n} \end{bmatrix} \quad (2)$$

So, each coefficient  $w_i$  is obtained by the formula (2):

$$w_i = \frac{\sum_{l=1}^n \frac{a_{il}}{\sum_{k=1}^n a_{kl}}}{n}. \quad (3)$$

And the sum of  $w_i$  must be equal to 1.

The result of this operation provides the percentages of the relative total priorities.

These calculations can be carried out by the **Expert Choice** software.

*Illustrative example*

Consider a decision problem of determining an «index (main objective) composite sustainable development» judging business performance. This index is divided into three sub-indexes (economic, environmental and social), which we refer to as secondary objectives. Suppose that they evolve in the following sense:

1<sup>st</sup> Economic > 2<sup>nd</sup> Environment > 3<sup>rd</sup> Social, so that the matrix of the judgments takes the form below (table 2).

The weights of the different objectives is obtained by normalizing the matrix.

Table 2

**Matrix of judgments**

Criteria	Economic	Environmental	Social	Env.	Tech.	Soc.	$\Sigma =$	$\Sigma / 3$
<b>Economic</b>	1	2	3	0.55	0.57	0.50	1.62	0.54
<b>Environmental</b>	1/2	1	2	0.27	0.29	0.33	0.89	0.30
<b>Social</b>	1/3	1/2	1	0.18	0.14	0.17	0.49	0.16
$\Sigma =$	1,83	3,5	6	1.00	1.00	1.00		

The economic criterion is most important with a weight equal to 54%, followed by the environment criterion with a weight of 30%. Therefore the second most important criterion then comes in last position. The social criterion with a weight of 16% – is the criterion of less importance.

*Approach based on the analysis of the eigenvalues*

A matrix with positive values admits a largest eigenvalue (in modulus)  $\lambda_{\max}$  that is unique (of multiplicity 1) and positive real. The eigenvector corresponding to it is determined a multiplicative factor close and its components have the same sign (Perron Frobenius theorem) [20]. For a comparison matrix weakly incoherent Saaty proposes to adopt this eigenvector (associated with  $\lambda_{\max}$ ) as an approximation of weight set to be searched. The eigenvector in question is normalized so that the sum of its components equals to 1.

Let us resume our original matrix of judgments we note **A** (see Table 4):

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1/2 & 1 & 2 \\ 1/3 & 1/2 & 1 \end{bmatrix} \quad (4)$$

This matrix admits a largest eigenvalue is  $\lambda_{\max} = 3.0092 \approx 3$ . The corresponding eigenvector is:  $\mathbf{V} = (0.8468, 0.4660, 0.2525)$ . By normalizing the eigenvector we have:  $V_n = (0.5396, 0.2970, 0.1634)$  or  $V_n = (0.54, 0.30, 0.16)$ .

Thua, it is found that the two approaches led to the same result. It now remains to check the consistency of the matrix of judgments. In our study, we use the approach, based on the analysis of the eigenvalues - because it appears less constraining to us and fast.

#### 2.4. Checking the consistency of results.

Method AHP makes it possible to measure the coherence of the comparisons and the intuitive choices, made by the decision makers [21]. Among the methods, used to determine the weights, AHP is the only one that allows such an audit to ensure that judgments made by the decision makers are not arbitrary. This approach is crucial since the experts can make mistakes in the assessments. This is the reason why the coherence measure is critical to detect these types of errors, which can greatly affect the results of the final analysis. The process of calculating the ratio of consistency varies depending on the approach used.

##### *Case where the normalization method is used*

The first step of calculating the ratio of coherence involves taking the initial matrix of judgments, i.e. that of the input data, and to multiply it by the vector priority (weights). It is then necessary to make the total of the values for each line of the new matrix. The resulting vector of this operation, i.e. consisting of the sum of each line, will be divided by the value of the priority vector associated therewith. The mean of the elements of this last vector obtained is represented by  $\lambda_{\max}$  (the greatest eigenvalue). All these operations are described by equations (2, 3 and 4).

We define the vectors  $[\lambda'_1 \dots \lambda'_i \dots \lambda'_n]$  et  $[\lambda_1 \dots \lambda_i \dots \lambda_n]$  such that:

$$\begin{bmatrix} \lambda'_1 \\ \dots \\ \lambda'_i \\ \dots \\ \lambda'_n \end{bmatrix} = \sum_{k=1}^n w_k \times \begin{bmatrix} a_{1k} \\ \dots \\ a_{ik} \\ \dots \\ a_{nk} \end{bmatrix} = w_1 \times \begin{bmatrix} a_{11} \\ \dots \\ a_{i1} \\ \dots \\ a_{n1} \end{bmatrix} + \dots + w_i \times \begin{bmatrix} a_{1i} \\ \dots \\ a_{ii} \\ \dots \\ a_{ni} \end{bmatrix} + \dots + w_n \times \begin{bmatrix} a_{1n} \\ \dots \\ a_{in} \\ \dots \\ a_{nn} \end{bmatrix} \quad (5)$$

and

$$\lambda_i = \frac{\lambda'_i}{w_i}. \quad (6)$$

Then we get:

$$\lambda_{\max} = \left[ \sum_{i=1}^n \lambda_i \right] / n. \quad (7)$$

The consistency index  $IC$  is then:

$$IC = (\lambda_{\max} - n) / (n - 1), \quad (8)$$

where  $n$  – number of criteria or sub-criteria considered.

To calculate the ratio of coherence index is divided by a consistency value  $IA$  (index random matrix of the same dimension) depending on the number of objectives given (Table 3).

$$RC = \frac{IC}{IA}. \quad (9)$$

Table 3

Random table indexes [21]									
3	4	5	6	7	8	9	10	11	12
0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.53

However, when the matrix is perfectly consistent, the maximum eigenvalue is equal to the dimension:



$\lambda_{\max} = n$  and  $IC = 0$ . In the case of an incoherent matrix was, one has:  $\lambda_{\max} > n$ . The overall consistency of assessment is assessed by the ratio of consistency  $RC$ . For Saaty, the value of latter must be at least equal to 10% (tolerance threshold). In case this value exceeds 10%, the appreciations may demand certain revisions. Table 4 indicates the ranges of values of acceptable consistency ratio.

Table 4

Table ratios acceptable consistency [14]

Matrix size (n)	3	4	5 and >
Ratios acceptable consistency	0.05	0.08	0.10

#### *Illustrative example*

Let us resume our original matrix of judgments. Table 5 allows to check the consistency of judgments made by experts.

Table 5

Example checking the consistency of judgments

Criteria	Economic	Environmental	Social	Weight [W]	[B]=[A]*[W]	[C]=[B]/[W]
<b>Economic</b>	1	2	3	0.54	1.62	3
<b>Environmental</b>	1/2	1	2	0.89	0.89	2.97
<b>Social</b>	1/3	1/2	1	0.49	0.49	3.06
$\Sigma =$						<b>9.03</b>

$$\lambda_{\max} = 9.03 / 3 = 3.01$$

$$IC = (3.01 - 3) / (3 - 1) = 0.005$$

$$RC = 0.005 / 0.58 = 0.0086$$

$RC < 0.1$  – then the matrix of judgments is consistent.

#### *2.5. Case where the approach based on the analysis of the eigenvalues used.*

When this approach is used, we calculate directly the consistency index using relation (5) then one goes back to the ratio of coherence by using the relation (6).

The AHP method has advantages that are worth highlighting. Indeed, the method is easy to use. The consistency of the whole of the comparisons is checked. Moreover, software using this method, exist: **Expert Choice** or **MultCSync** [24] for example. Lastly, it helps to understand the complexity of the real world [4]. However this method is not without disadvantages. Indeed, the choice of scale ranging from 1 (same important) to 9 (absolutely more important) is not justified mathematically. Moreover, the comparison time increases more rapidly than the number of criteria: equation (10) below shows the number of comparisons required  $N_{comp}$  in Depending on number of weight  $p$  to determine [6].

$$N_{comp} = \frac{n!}{2 \times (n-2)!} = \frac{1}{2} \times n \times (n-1) \quad (10)$$

**3. Direct weighting.** In the same manner it is possible to evaluate the performance of an action on a criterion directly - the expert can fix directly the weight criteria. This is called «direct weighting». This manner of making is less troublesome in this case, insofar as the weight does not need to be justified. However, experience shows that the weights are defined as generally not correspond to the meaning of the weights in the weighted average.

**4. Fixed point scoring.** The method of scoring grid, also better known by its original name of «scoring method» is an approach often used in multicriteria analysis. It comes to assigned a relative to each of the evaluation criteria for a total of 100% and thereafter weight, write down all the

possible options on each of the respective criteria. An ordinal scale previously set, allows the user to scoring. The order of options is then obtained by calculating the weighted sum for each option evaluated [2, 15]. The advantage of this method lies in its simplicity. Moreover, the attribution of a more important weight to a criterion reduces the relative importance of another element. For cons, the award of the relative weight of the criteria, is purely arbitrary, and the use of the ordinal scale in to rate the criteria to prevents a relative comparison between the different options. As an example, a score of 20 is not necessarily twice as good as 10 [9]. In a nutshell, this method is characterized by the difficulty of understanding the overall complexity of reality.

**5. Determination of weights in the MACBETH method.** Method MACBETH (Measuring Attractiveness by Categorical Based Evaluation Technique) was developed in the middle of the 90s by Bana e Costa and Vansnick, (2005) [3]. It is supported by an **M-MACBETH** software, developed by the authors of the method, which facilitates its usage. This method consists in helping a scholar to build in an interactive way a multicriteria function of values of additive form. To be done, she proposes, initially, to estimate for each criterion separately the marginal values of a set of actions on a scale from 0 to 100. These values are interpreted in terms of attractivity of the actions. In fact, the decision maker compares all equity criteria in pairs, on a purely ordinal scale of attractiveness. In a second step, the method proposes the same pattern of questions to compare the criteria themselves and derive their weight. As we are finding, the MACBETH method compares several steps and determination of weights is only one step. MACBETH allows to translate the semantic stated judgments by a decision maker on a numerical scale. Let  $S$  be a finite set of actions and  $G$  preference relations to measure the attractiveness between the elements of  $S$  ( $\forall x, y \in S, x G y$  if and only if the adjudicator judges that  $x$  is more attractive than  $y$ ). The MACBETH method is indeed an interactive procedure is to ask the maker to verbally judge the difference of attractiveness between the two actions  $x$  and  $y$  of  $S$  (with  $x$  is more attractive than  $y$ ) using semantic categories with an ordinal scale (see Table 6).

Table 6

**Degrees of attractiveness between alternatives**

Level of attractiveness	Difference in attractiveness	Semantic scale ( $k_0$ )
$C_0$	Null	$k_0 = 0$
$C_1$	Very weak	$k_1 = 1$
$C_2$	Weak	$k_2 = 2$
$C_3$	Moderate	$k_3 = 3$
$C_4$	Strong	$k_4 = 4$
$C_5$	Very Strong	$k_5 = 5$
$C_6$	Extreme Strong	$k_6 = 6$

During this interactive process, a matrix of categorical judgments will be built. For example, if the decision maker evaluates six shares **A, B, C, D, E** and **F** according to the quality criteria, we will have a matrix like the one, shown below (see Table 7).

Table 7

**Expert judgements for weight determination**

Quality	A	B	C	D	E	F
<b>A</b>	Null	Weak	Moderate	Moderate	Very Strong	Extreme
<b>B</b>		Null	Weak	Weak	Very Strong	Extreme
<b>C</b>			Null	Very Weak	Strong	Very Strong
<b>D</b>				Null	Strong	Very Strong
<b>E</b>					Null	Moderate
<b>F</b>						Null

In Table 6, we see that MACBETH uses a 7-level scale: the intensity of preference may be: «null», «very weak», «weak», «moderate», «strong», «very strong», «extreme». In case of insufficient knowledge, MACBETH expresses an ordinal preference denoted «P» or an unknown preference, denoted «?». As part of this work, we present a simplified case of determining weights in MACBETH.

One thus defines two reference levels for each of the criteria that will serve as anchors. The first, called «neutral » or « acceptable » level, is a level of performance below which significant research efforts should be made to allow the adoption of the solution. The second, called «good» or «satisfactory» level, is a level of performance - beyond which research to improve the performance of the solution, according to the criterion in question is no longer a priority. Using these reference levels is central to this method because it provides a valuation model, giving both absolute information (positioning solutions compared to baselines) and information relating (ranking solutions for each criterion).

Either the matrix of comparison of a set of 3 criteria (see table 8).

Table 8

Expert judgements for weight determination					
	Good	A	B	C	Neutral
Good	0	P	P	P	P
A		0	$k_1$	P	P
B			0	$k_2$	P
C				0	$k_3$
Neutral					0

From these preferences such  $k_i$  ( $k_i \in \{0, 1, 2, 3, 4, 5, 6\}$ ), it is possible to determine the weights  $\omega_i$  of different criteria.

Reading the matrix allows to write the following relations:

$$\begin{cases} 1 - \omega_i & > 0 \\ \omega_1 - \omega_2 & = \alpha k_1 \\ \omega_2 - \omega_3 & = \alpha k_2 \\ \omega_3 - 0 & = \alpha k_3 \\ \omega_1 + \omega_2 + \omega_3 & = 1 \end{cases} \quad (11)$$

Where  $\omega_1, \omega_2, \omega_3$  represent respectively the weights of the criteria **A, B, C**;  $\alpha$  is a real coefficient, which allows to respect the boundaries of the criterion area (the interval  $[0, 1]$  in our case);  $k_i$  represents the strengths of preference. Note that the good situation and the neutral situation are associated respectively to the vector of elementary performance expression (1, 1, 1) and (0, 0, 0) – i.e., after normalization. Therefore, if we consider that these two situations are formed from upper and lower bounds of quality attributes values, all other situations will be classified between them. Thus **A, B, C** can be represented respectively by (1, 0, 0), (0, 1, 0) and (0, 0, 1). By reorganizing equations (see system 11), we obtain the following result:

$$\left\{ \begin{array}{lcl} \omega_1 & = & \alpha (k_1 + k_2 + k_3) \\ \omega_2 & = & \alpha (k_2 + k_3) \\ \omega_3 & = & \alpha k_3 \\ \omega_3 - 0 & = & \alpha k_3 \\ \omega_1 + \omega_2 + \omega_3 & = & 1 = \alpha (k_1 + 2k_2 + 3k_3) \end{array} \right. . \quad (12)$$

In finding that:

$$\alpha = \frac{1}{k_1 + 2k_2 + 3k_3} \quad (13)$$

one has then :

$$\omega_1 = \frac{k_1 + k_2 + k_3}{\alpha} = \frac{k_1 + k_2 + k_3}{k_1 + 2k_2 + 3k_3}, \quad (14)$$

$$\omega_2 = \frac{k_2 + k_3}{\alpha} = \frac{k_2 + k_3}{k_1 + 2k_2 + 3k_3}, \quad (15)$$

$$\omega_3 = \frac{k_3}{\alpha} = \frac{k_3}{k_1 + 2k_2 + 3k_3}. \quad (16)$$

We can extend this reasoning to N criteria (or expressions of performance vectors) and in this case, one obtains the coefficient  $\alpha$  and the weights  $\omega_i$  arranged by decreasing order thanks to the following formulas:

$$\alpha = \sum_{j=1}^n \frac{1}{j * k_j} \text{ and } \omega_i = \alpha * \sum_{j=1}^n k_j = \frac{\sum_{j=1}^n k_j}{\sum_{j=1}^n j * k_j} \quad (17)$$

Method MACBETH of comparison per pair allows to perform a comparison of alternatives – determination of the weights is only one stage. This method is part of the most methods of used analysis. With few exceptions, the MACBETH method has the same advantages and disadvantages as the AHP method. But the big difference between these methods lies in the comparison mode. MACBETH is based on a comparison by difference, for example «X is better of four points than Y». Whereas for the AHP, the established user of the ratios between the options, for example « X is three times more important than Y ». The MACBETH method allows to combine two types of information: information on preferences, revealed by the decision maker; information on the importance of the criteria and their interactions, restricted to pairs of criteria. Nevertheless, the method presents some disadvantages: the scales of MACBETH method are not limited; in the method, the determination of the constants of scale follows the same process as for the determination of the attractivity scales.

The methods of weighting are not limited to those, we have presented in this article. In the literature there are others methods such as: expected value method [11]; tree of weightings [5]; method GRAMP; method based on the analysis of surfaces of equal global satisfaction [7] etc.

**Conclusion.** The methods of weights determination comprise the advantages and disadvantage in their applications and differ according to the usage need. However, they allows the decision maker all help to make a wise choice and better selection. Mindful not to have swept all methods of

weights determination, this study will remain to deepen, possibly including other methods. The objective is not to arrive at a single method that will solve all the problems, but rather to determine a list of more appropriate methods for each problem. Then depending on the question that we are facing, choose the method most appropriate. Finally, the main parameters, defining the usage of one method over another are simplicity, usage clarity and adaptability. Also, the choice of a method for determining weights must depend on experience feedback.

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### **СИСТЕМНЫЙ АНАЛИЗ НАПРАВЛЕНИЙ И ОСОБЕННОСТЕЙ ИНФОРМАТИЗАЦИИ СФЕРЫ ЗДРАВООХРАНЕНИЯ РОССИИ**

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Обоснована актуальность темы статьи с позиций обеспечения доступности и качества медицинской помощи для населения. Авторами рассмотрены общие особенности деятельности сферы здравоохранения (СЗ) России на современном этапе социально-экономического развития страны, включая факторы, обуславливающие конкуренцию между медицинскими учреждениями (МУ). Предложена классификация для иерархических уровней информатизации для СЗ. Показаны преимущества комплексного подхода к информатизации страны в целом, регионов, населенных пунктов. Обоснованы объективные и субъективные причины, тормозящие процессы осуществления такой информатизации. Подробно рассмотрены цели, направления и особенности информатизации деятельности отдельных МУ. Используемые в МУ программные средства (ПС) разделены на две группы: общего характера и специализированные, которые отражают специфику работы МУ. Для ПС общего характера охарактеризованы типичные для МУ классы программного обеспечения, сделаны оценки необходимой информационно-коммуникационной компетентности их пользователей. Предложена трехуровневая иерархическая структура для специализированных ПС, применяемых в МУ. Подробно рассмотрен состав медицинских информационных систем в различных типах медучреждений, информационные взаимосвязи между отдельными компонентами таких систем. Исследованы риски информационной безопасности при эксплуатации таких систем, возможные меры по снижению рисков.

**Ключевые слова:** сфера здравоохранения, качество медицинских услуг, информатизация, уровни, структура программных средств, медицинские информационные системы, информационная безопасность, высокотехнологичная медпомощь, DICOM-устройства, телемедицинские технологии